**MMAT5010 Linear Analysis (2024-25): Homework 5**
**Deadline: 8 Mar 2025**

# Important Notice:

♣ The answer paper must be submitted before the deadline.
♠ The answer paper MUST BE sent to the CU Blackboard.

1. Let $c_{00}$ be the finite sequence space. Assume that $c_{00}$ is endowed with the $\|\cdot\|_\infty$-norm. Define a linear map $T : c_{00} \to c_{00}$ by $Tx(k) := \frac{1}{k}x(k)$ for $x \in c_{00}$ and $k = 1, 2....$

   (a) Find $\|T\|$.

   (b) Show that $T$ is a linear isomorphism, that is $T$ is a linear bijection, and the inverse $T^{-1}$ is unbounded.

2. Let $(X, \|\cdot\|)$ be a normed space. Assume that $T : X \to X$ is a linear isomorphism.

   (a) For each $x \in X$, put $\|x\|_0 := \|Tx\|$. Show that $\|\cdot\|_0$ is a norm on $X$.

   (b) Show that the inverse $T^{-1}$ is bounded if and only if there is $c > 0$ such that $c\|x\| \le \|Tx\|$ for all $x \in X$.

   (c) The norms $\|\cdot\|$ and $\|\cdot\|_0$ are equivalent if and only if $T$ and $T^{-1}$ both are bounded.

**NOTE:** Question 1 (b) above shows us that a linear isomorphism $T$ is bounded but the inverse $T^{-1}$ may not be bounded in general.

$* * *$ **End** $* * *$